

Summary for Calculus 1 and Algebra

Robert Heß, January 16th 2012

1 Summary for Basics

1.1 Logic, sets and functions

- logic, compound statements, laws on logic, quantifiers
- sets, subsets, equal- and empty sets, union, intersection, complement, properties
- Cartesian product, relation, function, image-set
- injective, surjective and bijective functions, inverse functions

1.2 Natural numbers, and integers

- natural numbers and its properties
- prime number, prime factorization, gcd, lcm, product of gcd and lcm
- remainder, Euclidean algorithm
- number systems, conversion between them
- complete induction
- integers and its properties

1.3 Rational and real numbers

- rational numbers and its properties
- real numbers and its properties
- intervals, supremum, infimum, maximum, minimum, absolute value

1.4 Complex numbers

- imaginary unit j , imaginary number, powers of j , complex numbers, properties
- real- and imaginary part, absolute, polar form, argument
- basic arithmetic operations, conjugation, powers and roots of complex numbers

2 Summary for Calculus 1

2.1 Sequences

- sequence, bounded sequence, monotonicity, convergence, divergence
- limit of a sequence, calculating with limits

2.2 Series

- partial sum, series
- convergence, divergence, calculating with series,
- geometric sums, geometric series
- absolute convergence, comparison test, root test, ratio test, alternating series test

2.3 Power-series

- definition, convergence, main theorem on power series, radius of convergence
- exponential function, trigonometric functions, Euler's formula, addition theorem
- tangent, cotangent, values of sin, cos, tan and cot in 30° and 45° steps

2.4 Functions

- definition, monotonicity, bounded function, symmetry, periodicity, zeros, extrema
- composition function, inverse function
- (one sided-) limit, continuity, intermediate value theorem

2.5 Differential calculus

- differentiability, differentiability and continuity
- some important derivatives (table 9.1)
- basic operations of derivatives, chain rule, mean value theorem

2.6 Polynomials and rational functions

- polynomials, properties
- zeros, multiple zeros, fundamental theorem of algebra, complex zeros
- rational function, zeros, poles, l'Hôpital's rule
- partial fraction decomposition (real and complex poles)
- Taylor polynomial, propagation of uncertainty

2.7 Curve sketching

- domain, codomain, image, boundary
- derivatives and monotonicity, symmetry, periodicity
- intersection with axes, derivatives and extrema
- derivatives curvature, (removable) discontinuities, asymptotes
- drawing a sketch

3 Summary for Algebra

3.1 Boolean Algebra

- laws on boolean functions
- disjunctive and conjunctive normal form

3.2 System of linear equations

- Gauss-Jordan elimination
- reduced row echelon form
- solution behaviours of a homogeneous and inhomogeneous SLE

3.3 Matrices

- zero-, square-, diagonal-, identity- and triangular-matrix, unit-vectors
- addition, multiplication with scalar, transpose
- matrix multiplication, calculation rules
- multiplication with vector, product of column- and row-vectors
- matrices and SLE
- matrix inversion, properties, calculation rules
- linear independence, rank of a matrix
- rank of a matrix and influence on solution behaviours

3.4 Vector-space

- definition, vector-space of row-vectors and column-vectors, subspace
- linear combination, linear independence, span, spanning set, basis, dimension
- linear mapping, kernel, image, dimensions
- matrices as linear map, rank and dimension of image
- Solution space of an SLE, dimensions and rank

3.5 Determinants

- 2×2 and 3×3 matrices, rule of Sarrus, geometric interpretation
- Laplace expansion, properties, determinant of special matrices
- Cramer's rule