Surname: ..... Forename: ..... MatrNo.: .....

Exam: Mathematics 1

Hamburg University of Applied Science
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering
Prof. Dr. Robert Heß, 17.7.2015, duration: 90 Min.
Permitted aids: up to six A4-pages of personal notes (i.e. single sided sheets)

Result: ..... of 100 points Mark: ..... points.

Problem 1 (18 points)

Prove by mathematical induction: $1 + \sum_{k=1}^{n} \frac{2^{2k-2}}{3^k} = \left(\frac{4}{3}\right)^n$

Problem 2 (15 points)

Find all solutions in Cartesian form for $z \in \mathbb{C}$ with $z^4 = -4$.

Problem 3 (12 points)

Resolve, i.e. differentiate the following expressions:

$$a = \frac{\mathrm{d}}{\mathrm{d}x} \sin(2x)\cos(3x) \qquad b = \frac{\mathrm{d}}{\mathrm{d}y} \exp\left(\sin(x) + 3^y + t^3\right) \qquad c = \frac{\mathrm{d}^n}{\mathrm{d}t^n}\, \hat{y}\, e^{\mathrm{j}2\pi f t}$$

Problem 4 (15 points)

Evaluate and sketch the region of convergence of the power series: $f(z) = \sum_{k=0}^{\infty} \frac{(z+2\mathrm{j})^k}{3^k}$, $z \in \mathbb{C}$

Problem 5 (20 points)

Analyse the function $f(x) = \frac{1}{2}x^4 - 2x^3 + 8x - 3$ with respect to inflection and saddle points.

Problem 6 (20 points)

For the following SLE determine
a) the reduced row echelon form of the extended coefficient matrix,
b) the rank of the coefficient matrix and
c) the rank of the extended coefficient matrix.
d) Draw your conclusion on the solution behaviour by the derived ranks.

$$x + 2y + 3z = 1 \qquad x + y + z = 1 \qquad 2x + 2y + z = 1 \qquad 2x + y + z = 1$$